Real Option Value, Chapter 8 Scale

Appendix B Uncertain Abandonment Value

The opportunity to terminate a project represents a managerial option that is embedded in an operating asset, which is exercised whenever the expected present value of residual operating cash flows is sufficiently lower than the value of abandoning the project. The abandonment opportunity creates additional project value as soon as the investment commitment is made, but the traditional literature on scale options usually assumes the abandonment value is known and constant.

The importance of a stopping event for a deterministic capital budgeting model is possibly first raised in a series of analytical studies on depreciation and replacement, Preinreich (1938, 1939, 1940). Within a deterministic framework, the role of abandonment as a valuable source of cash flow having the potential to modify the present value and alter the investment policy is analyzed by Robichek and Van Horne (1967). Ignoring abandonment excludes the flexibility value due to the funds that are released. By allowing the abandonment timing to be variable, Dyl and Long (1969) show this flexibility is a source of additional value. While Gaumitz and Emery (1980) extend this formulation, to establish that the presence or absence of asset replacement impinges directly on the abandonment decision, Howe and McCabe (1983) develop a more comprehensive model involving replacement and abandonment.

Using a real option framework, McDonald and Siegel (1985) and Myers and Majd (1990) establish that abandonment can be represented as a put option and that the project life is not fixed but determined by the decision to abandon. The impact of abandonment on the investment decision is also studied for particular contexts. As a way of retaining a single factor formulation, Mauer and Ott (1995) represent abandonment value as a function of operating cost, while Dobbs (2004) deduces the abandonment value from the operating cost threshold. Paxson (2005) incorporates abandonment in a real asset option model, assuming fixed negative abandonment values. The operating cost and abandonment value are treated as two distinct factors by Adkins and Paxson (2010) in order to analyze their interaction in making replacement decisions.

Immediately following initial investment expenditure, the firm possesses an abandonment option as a direct consequence of operating the asset. Exercising this option enables the firm to exchange the operating project for its abandonment value. This presumes that the project as an asset can be liquidated in some way either because its physical value can be captured through the second-hand or scrap-metal markets, by transferring the asset to an alternative geographic region, or by selling the embedded technological or know-how knowledge. For an operating project, a performance assessment can form the basis for deciding whether it should be continued or abandoned, and the choice is predicated on the relative prevailing magnitudes of the project value and the abandonment option value.

There are two stochastic factors specified in this model, abstracted from Adkins and Paxson (2014), denoted by V and X, representing the remaining project present value, and the abandonment value, respectively. Each of the two factors is described by a geometric Brownian motion process (gBm) with drift. If Ψ denotes a generic factor with $\Psi \in \{\Psi_1, \Psi_2\} = \{V, X\}$, then:

$$\mathrm{d}\Psi = \alpha_\Psi \Psi \mathrm{d}t + \sigma_\Psi \Psi \mathrm{d}z_\Psi , \tag{1}$$

where α_Ψ denotes the instantaneous drift term per unit of time, σ_Ψ the instantaneous volatility per unit of time, and $\mathrm{d}z_\Psi$ is an increment of the standard Wiener process. Dependence amongst the two stochastic factors is described by the instantaneous covariance term $\rho_{ij}\sigma_i\sigma_j$ for $i, j = 1, 2; i \neq j$, where $\mathrm{Cov}\left[\Psi_i, \Psi_j\right] = \rho_{ij}\sigma_i\sigma_j\Psi_i\Psi_j\mathrm{d}t$ and $\left|\rho_{i,j}\right| \leq 1$, $i, j = 1, 2; i \neq j$.

Formulating the two factors of interest according to a gBm process has the merit of generating solutions consistent with other real option models. However, it does entail recognizing that the values adopted by each factor are confined to the positive domain. While this assumption is plausible for the project value, the same cannot be said for the abandonment value. There are circumstances, such as the scrap metal value for retired ships and plant & equipment either sold to third parties or exported abroad, that support the assumption, but there are others, such as the decommissioning payments required for a redundant nuclear power station or the costs of decontaminating a brown-field site where the abandonment value is clearly negative. This model deals only with positive abandonment values.

Denote the abandonment option value generically by F, so $F = F\left(V, X\right)$. By applying Ito's lemma to (1), the valuation relationship for F is specified by:

$$\begin{aligned} &\tfrac{1}{2}\sigma_V^2 V^2 \frac{\partial^2 F}{\partial V^2} + \tfrac{1}{2}\sigma_X^2 X^2 \frac{\partial^2 F}{\partial X^2} \\ &+\rho_{VX}\sigma_V\sigma_X VX \frac{\partial^2 F}{\partial V \partial X} + \theta_V V \frac{\partial F}{\partial V} + \theta_X X \frac{\partial F}{\partial X} - rF = 0. \end{aligned} \tag{2}$$

where the parameters θ_V and θ_X denote the respective risk neutral drift terms[1], and r the risk-free rate. By extension, see Adkins and Paxson (2011), McDonald and Siegel (1986), a product power function involving the two factors V and X can be shown to be the solution to the two dimensional valuation relationship. The generic valuation function for the abandonment option at stage 1 is:

$$F_1\left(V, X\right) = A_1 V^{\beta_1} X^{\phi_1}, \tag{3}$$

where A is a generic coefficient, and β and ϕ are the respective generic power parameters for V and X. While $A > 0$, since an option value is always non-negative, the power parameters can be of either sign contingent on the particular context. The option value (3) satisfies the valuation relationship with characteristic root equation Q:

$$\begin{aligned} Q\left(\beta, \eta, \phi\right) = &\tfrac{1}{2}\sigma_V^2 \beta\left(\beta - 1\right) + \tfrac{1}{2}\sigma_X^2 \phi\left(\phi - 1\right) \\ &+ \rho_{VX}\sigma_V\sigma_X \beta\phi \\ &+ \theta_V \beta + \theta_X \phi - r = 0. \end{aligned} \tag{4}$$

In this model, the available abandonment opportunity arises after investment, that is it emerges as a consequence of exercising an original project opportunity option and making the investment commitment. When investigating the justification for an abandonment, we treat the previous investment expenditure as a sunk cost, since it exerts no influence over the decision to abandon and plays no role in determining the abandonment option value. We assume once abandoned there is no subsequent investment opportunity. This is appropriate for a bankrupt firm, or where X is far below K and any subsequent investment funding is problematical. Instead, the abandonment choice is decided by the prevailing levels of the present value for the project and the value obtained through abandonment. Although sunk, the investment cost is not completely

[1] Some authors assume θ=r-α, without a risk adjustment. It is likely that these drifts may be related for some types of equipment such as cars, but not perhaps for ships, but we ignore these possibilities.

irrecoverable, since the expenditure may be partially reimbursed through the receipt of the abandonment value.

Abandonment is justified whenever the prevailing value for V is sufficiently low while that for X is sufficiently high, since the firm would have to be convinced of the expected net benefits accruing from sacrificing the operating project value for the abandonment value. Moreover, the motivation justifying an abandonment intensifies and the corresponding option value increases as V continues to decline or X to rise. This suggests that F_1 is a monotonic increasing and decreasing function of V and X, respectively, and entails that $\beta_1 < 0$ and $\phi_1 > 0$.

Owing to value conservation, abandonment is economically warranted when the composite asset values just prior and after exercise are in balance. Just prior to exercise, the value is composed of the sum of the project present value and the abandonment option value. At the instant of exercise, this composite amount is being sacrificed to acquire the benefit of the abandonment value. If the threshold levels signalling exercise are denoted by $\hat{V}_1$ and $\hat{X}_1$ for the project present value and the abandonment value, respectively, then the composite asset value just prior to exercise is specified by $\hat{V}_1 + F_1\left(\hat{V}_1, \hat{X}_1\right)$, and the asset value just after exercise by $\hat{X}_1$. It follows that the value matching relationship is defined by:

$$\hat{V}_1 + A_1 \hat{V}^{\beta_1} \hat{X}^{\phi_1} = \hat{X}_1. \tag{5}$$

For an optimal exercise, the smooth pasting or first order conditions must be satisfied. Since there are two factors of interest, there are two smooth pasting conditions, one for each factor, V and X, respectively. These can be expressed as:

$$\hat{V}_1 + \beta_1 A_1 \hat{V}_1^{\beta_1} \hat{X}_1^{\phi_1} = 0, \tag{6}$$

$$\phi_1 A_1 \hat{V}_1^{\beta_1} \hat{X}_1^{\phi_1} = \hat{X}_1. \tag{7}$$

The conjecture $\beta_1 < 0$ and $\phi_1 > 0$ is corroborated by (6) and (7), respectively, since $A_1 > 0$. By inspecting (5)-(7), we conclude that $\beta_1 + \phi_1 = 1$, which implies that F_1 is a homogenous degree-1 function. The parameter β_1 is evaluated as the negative root solution to (4):

$$Q\left(\beta_1, 0, 1-\beta_1\right) = Q_1\left(\beta_1\right) = 0. \tag{8}$$

$$\hat{V}_1 = \frac{-\beta_1 \hat{X}_1}{1-\beta_1}, \tag{9}$$

$$A_1 = \frac{1}{-\beta_1}\left(\frac{-\beta_1}{1-\beta_1}\right)^{1-\beta_1}. \tag{10}$$

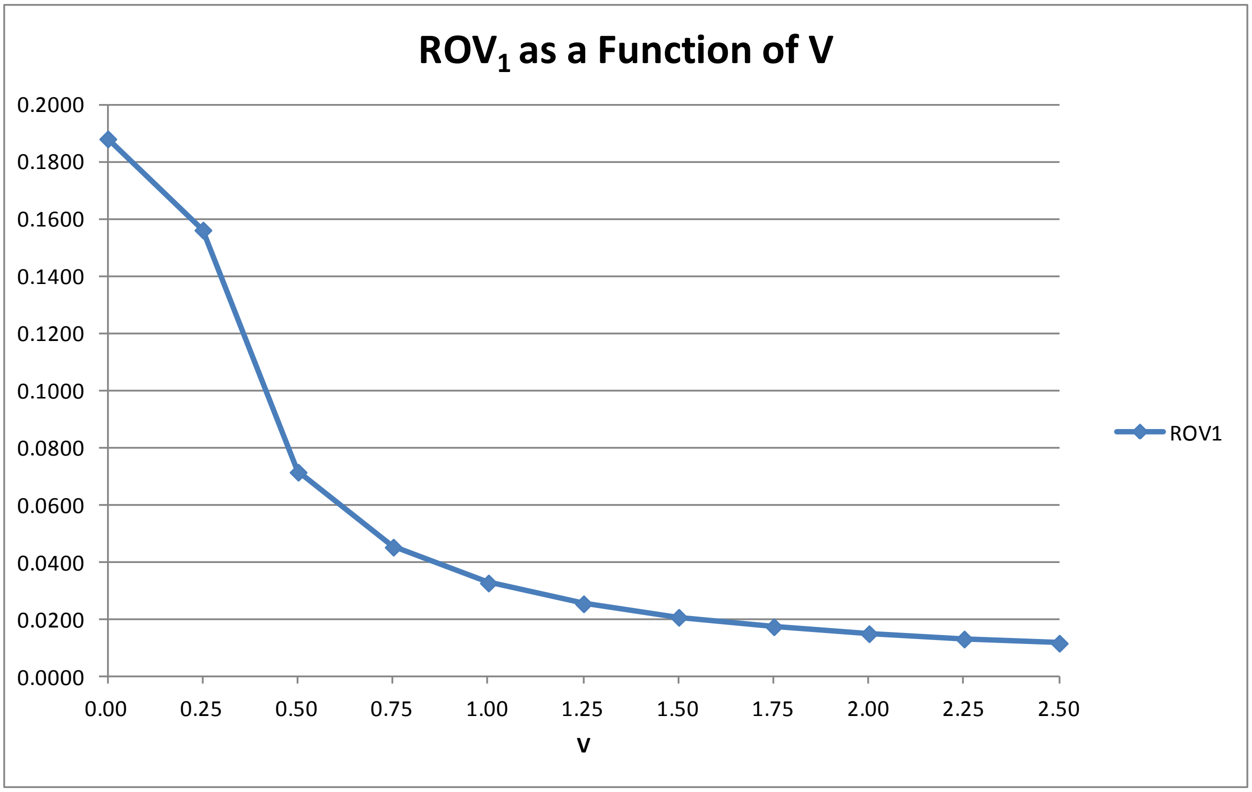
Figure 1

	A	B	C	D	E
1		**ABANDON OPTION VALUE**			
2	INPUT				
3	V	1.00			
4	X	0.40			
5	σV	0.20			
6	σX	0.10			
7	ρ VX	0.00			
8	r	0.06			
9	θV	0.00			
10	θX	0.00			
11	OUTPUT				
12	Q(β,ϕ)	0.0000	EQ 4+8	0.5*(B5^2)*B19*(B19-1)+0.5*(B6^2)*B18*(B18-1)+B7*B5*B6+B9*B19+B10*B18-B8	
13	SP1	0.0000	EQ 6	B20+B19*B17*(B20^B19)*(B21^B18)	
14	SP2	0.0000	EQ 7	B18*B17*(B20^B19)*(B21^B18)-B21	
15	VM1	0.0000	EQ 5	B17*(B20^B19)*(B21^B18)+B20-B21	
16	SOLVER	0.0000		Set B16=0, Changing B17:B20	
17	A1	0.2297	EQ 10	(1/-B19)*(-B19/(1-B19))^(1-B19)	0.2297
18	ϕ1	2.1279		1-B19	2.1279
19	β1	-1.1279			
20	V1*	0.2120	EQ 9	-B19*B21/(1-B19)	0.2120
21	X1*	0.4000			
22	ROV1	0.0327	EQ 3	IF(B3>B20,B17*(B3^B19)*(B4^B18),B21-B20)	
23	V^/X^	0.5300		B20/B21	

Figure 1 is a simple spreadsheet where equations 4-7 are set equal to zero, by changing the unknowns A_1, ϕ, β and V_1*, assuming X_1=X_1*. With the inputted parameter values, if X_1=.4, the abandonment is justified if V suddenly falls from 1.0 to below .21, or way below the abandonment value. The real option value ROV is very low at .03, since V1>>V1*.

A further numerical analyses in Figure 2 shows the abandonment option value as a function of V, keeping X and X* and other parameter values constant. The abandonment option value is greatest when V is lowest, or V<X at the thresholds, when the project should be abandoned. At high values of V, there is naturally little abandonment value as in Figure 1.

Figure 2



Further simulations using this model might consider large negative drifts for V (where there is physical deterioration), risk-adjusted and correlated drifts, technological obsolescence, and tax and regulatory incentives. Eventually there will be complex abandonment models, with other options such as second-hand sales and repurchases, and embedded options of stochastic contracting, expansion and stochastic operating costs. Finally, this model ignores multiple investment and abandonment opportunities, where the option holder might have a perpetual option to renew investments, or alternatively where there might be some probability of the option holder losing a perceived investment opportunity or being deprived of an abandonment option.

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